Modelos atmosféricos para a determinação de órbitas de satélites artificiais – resenha

Atmospheric models for artificial satelites orbit determination – a review

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Abstract

In this work we present a review of the most important models of the atmospheric density, as necessary for a precise evaluation of the aerodynamic forces acting on an artificial satellite of the Earth at altitudes above 50 km. All available modes in the literature are briefly described. Jacchia´s Model taking into account successive contributions by Harris, Priester e Roberts is developed in details. The development of Brazilian spacecrafts today and in near future and their orbital estimation and control justify this development.

Keywords

Atmosphere. Ionosphere. Models. Satellites.

Resumo

Neste trabalho apresentamos uma resenha dos mais importantes modelos da densidade atmosférica necessários para uma estimação precisa das forças aerodinâmicas que atuam na superfície de um satélite da Terra em altitudes acima de 50 km. Todos os modelos disponíveis na literatura são descritos em breve. O modelo de Jacchia levando em conta contribuições sucessivas de Harris, Priester e Roberts é descrito em detalhes. O desenvolvimento de artefatos espaciais brasileiros presentes e num futuro próximo com a necessária tarefa de estimação e controle de suas trajetórias justificam esse desenvolvimento.

Palavras Chave

Atmosfera. Ionosfera. Modelos. Satélites.

Introduction

The modeling of the aerodynamic force acting on a satellite in a near-earth orbit is a difficult task due to several reasons. The most relevant is that the characterization of the density above 150 km from the earth surface is extremely complex. The exact nature of the atmospheric density variations is not completely understood and only observational and experimental evidence indicate diurnal and seasonal variations, as well as effects due to changes in solar flux and geomagnetic activity. These effects can be modeled with some degree of confidence.

Neutral particle atmospheric drag is a significant non-conservative force modeling problem:

- for geodetic satellites which are normally deployed in the 750 to 1000 km altitude range,
- for new missions orbiting at low-Earth altitudes of between 325-450 km altitudes, and
- for satellites with complex shapes and large area to mass ratios.

Near-Earth satellites travel through a rarefied atmospheric medium. Atmospheric density varies with altitude and is highly dependent on solar heating and geomagnetic activity. The relative elemental constituents (N_2, N, O, O_2, H, He, Ar) also vary with height and geographical location further complicating accurate modeling of drag forces.

Atmospheric drag models commonly in use for these calculations include the DTM model (BARLIER et al., 1978) and the MSIS model (HEIDIN, 1987). The state of the current atmospheric density models for satellite drag modeling up to 1993 was reviewed by Marco et al. (1993) where the influence of surface properties was also evaluated. These models are based on either in situ atmospheric spectrometer measurements (MSIS) or satellite orbit dynamics (DTM). Although these models are extensively utilized, they suffer from incomplete global coverage, long time constants requiring a great deal of averaging, and aliasing from other unmodeled non-conservative forces for models estimated from satellite tracking data. These models are also undersampled at geodetic satellite altitudes (>750 km) and during times of high solar and/or geomagnetic activity where they produce density profiles based largely on extrapolations. Therefore, in order to achieve the accuracy needed for precision orbit determination, it is a common practice to solve for several drag scaling parameters to better model the observed satellite motion.

Until recently, the combination of satellite dynamical and in situ measurements had not been attempted; however, on-going model development is focused on producing combination solutions (CUNNINGHAM et al., 1994). This effort attempts to improve the MSIS model at higher altitudes through the inclusion of dynamically reduced geodetic satellite data. Nuth (1991) utilized the SLR tracking on Starlette and Ajisai in an attempt to improve the density modeling at geodetic satellite altitudes. A horizontal wind model (HEIDIN et al., 1991) along with accurate spacecraft attitude and non-conservative force models are being developed to decouple the drag density signal from other forces. Satellite data combined with atmospheric wind data could then be used to increase the accuracy of wind field models.

There remain several weaknesses in state-of-the-art drag modeling. Currently, no atmospheric wind effects are being considered within these models nor are they being applied externally in orbit drag computations. Even from what little is known about the mean wind fields at satellite altitudes, the assumption that the atmosphere rotates with the Earth is clearly invalid, especially towards the poles (BERGER and BARLIER, 1991). It is also a common practice to compute only the along track drag acceleration based on the computed projected area in the velocity direction. Out of plane drag forces are thereby neglected. Modeling these effects and/or estimating drag coefficients in the off-velocity directions can be used to further enhance modeling. These approaches are discussed by Ries et al. (1993a).

A complete and updated description of all major atmospheric models is found at NASA (2006). In this site we find actual on line evaluations of atmospheric and ionopheric density values and related properties.

The work of the U.S. Committee on Extension to the Standard Atmosphere (COESA), established in 1953, led to the 1958, 1962, 1966, and 1976 versions of the U.S. Standard Atmosphere. These models were published in book form jointly by the National Oceanic and Atmospheric Administration (NOAA), the National Aeronautics and Space Administration (NASA), and the U.S. Air Force. Altogether 30 U.S. organizations representing government, industry, research institutions, and universities participated in the COESA effort. Based on rocket and satellite data and perfect gas theory, the atmospheric densities and temperatures are represented from sea level to 1000 km. Below 32 km the U.S. Standard Atmosphere is identical with the Standard Atmosphere of the International Civil Aviation Organization (ICAO). The U.S. Standard Atmospheres 1958, 1962, and 1976 consist of single profiles representing the idealized, steady-state atmosphere for moderate solar activity. Parameters listed include temperature, pressure, density, acceleration caused by gravity, pressure scale height, number density, mean particle speed, mean collision frequency, mean free path, mean molecular weight, sound speed, dynamic viscosity, kinematic viscosity, thermal conductivity, and geopotential altitude. The altitude resolution varies from 0.05 km at low altitudes to 5 km at high altitudes. All tables are given in English (foot) as well as metric (meter) units. The U.S. Standard Atmosphere Supplements, 1966 includes tables of temperature, pressure, density, sound speed, viscosity, and thermal conductivity for five northern latitudes (15, 30, 45, 60, 75), for summer and winter conditions.

The Jacchia Reference Atmospheres were published as reports in 1970, 1971, and 1977. These publications include explanatory text, formulas, and tables. The density, temperature, and composition are listed in the altitude range 90 km to 2500 km. Variations with season, latitude, and local time are considered. Auxiliary tables are provided to evaluate geomagnetic, semi-annual, and seasonal-latitudinal effects. Jacchia's models are based mostly on satellite drag data. Assuming diffusive equilibrium, the atmospheric profiles are defined by the exospheric temperature. He contributed the thermospheric part (110 km to 200 km) to the CIRA-72 model. Jacchia (1964) was the first to point out the coupling between solar wind and atmosphere.

Atmospheric Handbook 1984:this data set was compiled by V. E. Derr and is available in hard copy and on magnetic tape. The parameters were collected over many years in response to requests by researchers for atmospheric electromagnetic wave propagation. Data presented include attenuation coefficients for the atmosphere and H2O; 1962 Standard Atmospheres; cloud drop size distributions for water and ice spheres; solar spectral irradiance; sky spectral radiance; Rayleigh coefficients for air; refractive indices for air, ice, liquid H2O, and various atmospheric aerosols; and relative reflectance for ice and H2O.

The Chiu Ionospheric Model 1975 is a global phenomenological model describing the large scale variations of ionospheric electron density with local time, latitude, and solar sunspot number. It is based on ionosonde data from 50 stations spanning the period 1957 to 1970. The model profile is obtained as the sum of three Modified Chapman functions for E-, F1-, and F2-layers. The model was improved by Chiu (1975) and served as the starting point for the FAIM model. The model is fairly simple, using less than 50 coefficients, which limits its application for equatorial and higher latitudes. It is, however, fast and easily manipulable and a good choice for first-order estimates. An extension for the polar cap ionosphere is being constructed.

Bent Ionospheric Model 1972 described the ionospheric electron density as a function of latitude, longitude, time, season, and solar radio flux. The topside is represented by a parabola and three exponential profile segments, and the bottomside by a bi-parabola. The model is based on about 50,000 Alouette topside ionograms (1962-1966), 6,000 Ariel 3 in situ measurements (1967- 1968), and 400,000 bottomside ionograms (1962-1969). For the F2-peak the CCIR maps are used. The model has been widely used for ionospheric refraction corrections in satellite tracking. It does not include the lower layers (D, E, F1) and uses a simple quadratic relationship between CCIR's M(3000)F2 factor and the height of the F2-peak. A comparison between the Bent model and the IRI Model and their application for satellite orbit determination was discussed by Bilitza et al. (1988). IRI showed better results because of the more detailed representation of the bottomside density structure.

An exospheric hydrogen density model was developed by Hodges (1994). In this model a Monte Carlo simulation of the terrestrial hydrogen exosphere is used to derive a global model of the exospheric hydrogen density. A third-order spherical harmonic expansion in longitude and colatitude is used to represent H at a particular radius. The h_exos.dat file provides the harmonic expansion coefficients for 40 radii (between 6640 km and 62126 km) for solstice and equinox conditions, and for four levels of solar activity (F10.7 = 80, 130, 180, 230). Details of the Monte Carlo simulation are explained in Hodges (1994). The simulation results show significant differences with previous exosphere models, as well as with the H distributions of the MSIS-86 thermosphere model.

The NRLMSIS-00 empirical atmosphere model was developed by Picone, Hedin and Drob (2002) based on the MSISE90 model. The main differences to MSISE90 are noted in the comments at the top of the computer code. They involve (1) the extensive use of drag and accelerometer data on total mass density, (2) the addition of a component to the total mass density that accounts for possibly significant contributions of O+ and hot oxygen at altitudes above 500 km, and (3) the inclusion of the SMM UV occultation data on [O2]. The MSISE90 model describes the neutral temperature and densities in Earth's atmosphere from ground to thermospheric heights. Below 72.5 km the model is primarily based on the MAP Handbook (LABITZKE et al., 1985) tabulation of zonal average temperature and pressure by Labitzke, Barnett and Edwards (1985), which was also used for the CIRA-86. Below 20 km these data were supplemented with averages from the National Meteorological Center (NMC). In addition, pitot tube, falling sphere, and grenade sounder rocket measurements from 1947 to 1972 were taken into consideration. Above 72.5 km MSISE-90 is essentially a revised MSIS-86 model taking into account data derived from space shuttle flights and newer incoherent scatter results.

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The Mass-Spectrometer-Incoherent-Scatter (MSIS) model describes the neutral temperature and densities in the upper atmosphere (above about 100 km). MSIS-86 constitutes the upper part of the COSPAR International Reference Atmosphere (CIRA) 1986. The MSIS model is based on the extensive data compilation and analysis work of A. E. Hedin and his colleagues. Data sources include measurements from several rockets, satellites (OGO 6, San Marco 3, AEROS-A, AE-C, AE-D, AE-E, ESRO 4, and DE 2), and incoherent scatter radars (Millstone Hill, St. Santin, Arecibo, Jicamarca, and Malvern). The model expects as input year, day of year, Universal Time, altitude, geodetic latitude and longitude, local apparent solar time, solar F10.7 flux (for previous day and three-month average), and magnetic Ap index (daily or Ap history for the last 59 hours). For these conditions the following output parameters are calculated: number density of He, O, N_2, O_2, Ar, H, and N, total mass density; neutral temperature and exospheric temperature. For diagnostic purposes the source code is equipped with 23 flags to turn on/off particular variations. Hedin (1988) compared all three MSIS models with each other and with the Jacchia 1970 and 1977 models.

The Marshall Engineering Thermosphere Model (MET) is essentially a modified Jacchia 1970 model that includes some spatial and temporal variation patterns of the Jacchia 1971 model. In addition to thermospheric densities and temperatures the well-documented code provides also several often used parameters like gravitational acceleration and specific heat. MET was developed at NASA's Marshall Space Flight Center in Huntsville primarily for engineering applications. The MSIS model is generally considered superior to MET because of its larger data base and its more elaborate mathematical formalism.

The COSPAR International Reference Atmosphere (CIRA) provides empirical models of atmospheric temperature and densities from 0 km to 2000 km as recommended by the Committee on Space Research (COSPAR). Since the early sixties different editions of CIRA have been published: CIRA 1961, CIRA 1965, CIRA 1972. The CIRA Working Group meets bi- annually during the COSPAR General Assemblies. In the thermosphere (above about 100 km) CIRA-86 is identical with the MSIS-86 model. For the lower part (0 km to 120 km) the model consists of tables of the monthly mean values of temperature and zonal wind for the latitude range 80 N to 80 S. Two sets of files are provided, one in pressure coordinates, including also the geopotential heights, and one in height coordinates, including also the pressure values. These tables were generated by Fleming et al. (1988) from several global data compilations including ground-based and satellite (Nimbus 5, 6, 7) measurements: Oort (1983), Labitzke et al. (1985). The lower part was merged with MSIS-86 at 120 km altitude. In general, hydrostatic and thermal wind balance is maintained at all levels. The model accurately reproduces most of the characteristic features of the atmosphere, such as the equatorial wind and the general structure of the tropopause, stratopause, and mesopause.

The CIRA software archived at NSSDC includes the original CIRA-86 data files in binary format as provided by E. Fleming (Nov 1989). A simple drive program was written at NSSDC to facilitate access to the binary data. In addition a corrected version of the CIRA data files as provided by J. Barnett in July 1990 is provided in ASCII format.

The Penn State Mk III model produces (1) tables of ionospheric electron densities from 120 km to 1250 km, (2) the ionospheric electron content, and (3) statistical properties of sporadic E occurrence. Two modes of operation are available. One generates two to 24 profiles throughout a day at one location, and the other generates a set of profiles at a range of locations at one universal time. The model combines theoretical computations with empirical models for the F-peak parameters. Mk III is an updated version of the earlier models developed at the Pennsylvania State University by Nisbet (1971) and Lee (1985). It uses the MSIS-83 atmospheric model, solar fluxes measured by the AE-E satellite (Hinteregger and Fukui, 1981), reaction rates from Torr et al. (1979), and the semi-empirical maps for the F-peak parameters developed by Rush et al. (1984). Meanwhile, most of these parameter models have been updated. For these we refer to URSI foF2 model Map (NASA, 2006) and the MSIS Model, and these updates should be incorporated into the model code.

Atmosphere and Aerodynamics Forces Modelling

According to Cappellari; Velez and Fuchs (1976) atmospheric density models can be described in at least two different points of view.A model can be characterized by its dependence on altitude and its independence of any other parameters. Another model can be characterized by its dependence not only on altitude, but also on the position of the sun relative to the earth and the amount of energy emitted from the sun.

To account for various geomagnetic and solar activities, several atmospheric models have been constructed over the past several years (JACCHIA, 1960, 1963, 1964, 1971; HARRIS and PRIESTER,1952, 1962, 1965; ROBERTS, 1971)..

The most important type of solar radiation in terms of the effect on the structure of the atmosphere, results from solar ultraviolet radiation; its effect on temperature and density is maximum two to three hours after local noon. This radiation heats the atmosphere by conduction and thereby increases the density at higher altitudes. The process is known as the diurnal (or day-night) effect and causes a redistribution of density, resulting in a diurnal bulge in the atmosphere.

A second type of solar activity affecting the atmosphere results from extreme ultraviolet radiation. The atmospheric oscillations that are in phase with this solar flux are often referred to as the erratic or 27-day variations, since the oscillations sometimes exhibit a semi-regular character for intervals of several months, during which a period of 27 days is easily recognizable. It has been found that the 10-cm flux from the sun apparently varies in the same manner as the extreme ultraviolet emission, and can therefore be used as a fairly reliable index of short-term solar activity.

The third type of radiation is corpuscular in nature and is referred to as the solar wind. It is responsible for the changes in intensity and energy spectrum observed in the cosmic radi-ation and is the largest single factor affecting short-term fluctuations in the atmospheric density. Experiments on board Pioneer V were the first to establish that the 11-year solar cycle is a phenomenon that is not localized near the earth or its immediate environment but rather affects large volumes of the inner solar system. The solar wind is modeled as a interplanetary plasma streaming radially and irregularly outward from the sun, compressing the earth's magnetic field on the sunward side and extending it on the night side.

Atmospheric oscillations connected with geomagnetic storms are of significant amplitude but of very short duration (one or two days). Present-day studies indicate a correlation of atmospheric density with geomagnetic activity. Apart from the difficulty of accurately representing the environment at the satellite position, the second aspect of the problem lies in the complica-tion of rigorously modeling the force itself as a function of spacecraft configura-tion and attitude.

The force per unit mass caused by the atmospheric drag may be written, as a first approximation, as

$$F_D = \frac{1}{2} C_D \frac{A}{m} \rho\, v^2$$

in direct opposition to the motion of the center of mass of the satellite. In this equation, C_D is the form coefficient, *A* the cross section area of the satellite normal to the velocity of the satellite relative to the atmosphere, *m* the mass of the satellite, *r* the atmosphere density and *v* the velocity of the satellite relative to the atmosphere.

Rigorous treatment of the aerodynamics of free molecular flow involves the representation of the complex interaction of the atmospheric molecules with the surface molecules of the spacecraft. Under certain conditions, this inter-action is characterized as a specular or perfectly elastic reflection of the impinging molecules. The reflection is termed diffuse when the impinging molecules penetrate the surface, experience multiple collisions with the body molecules, and are re-emitted randomly with no memory of their prior history. In the case of specular reflection, there is no momentum transfer, and hence no force, tangential to a local surface element. Diffuse reflection does result in such a component of force, although it is small. In general, both types of phenomena are involved to varying degrees, depending upon the details of surface reflectivity and emissivity, temperature, free-stream constituents and their mean molecular motion. Conditions typical of most actual situations result in forces which can be adequately represented in terms of the specular reflection equations. Therefore, force modeling in an ordinary orbit determination system makes this simplifying assumption, and computes the force acting on a local surface element as the momentum transfer normal to that element.

The forces on all elements of the spacecraft surfaces exposed to the free-stream must be resolved in some coordinate frame and summed in order to obtain the total aerodynamic force acting on the spacecraft. This resolution has been performed for a number of elemental shapes at various orientations. A force coefficient, C_D, is defined as the nondimensional quantity

$$C_D = \frac{2F_D}{\rho\ v^2 A} \tag{1.1}$$

where

$F_D =$ the magnitude of the force acting on the object

$\rho =$ the density of the medium through which the object is moving

the magnitude of the velocity of the object wrt the medium producing the force

the cross section of the satellite wrt the relative velocity or, in most practical cases, an arbitrary reference area

The velocity of the spacecraft relative to the atmosphere is determined in the inertial coordinate system by subtracting the motion of the atmosphere, assumed to rotate with the earth, from that of the spacecraft

$$\vec{v}_{rel} = \dot{\vec{r}} - \vec{\omega} \times \vec{r} \tag{1.2}$$

The earth rotation vector $\overline{\omega}$ must be appropriately defined in the inertial frame (mean equator and equinox of 1950.0 or true equator and equinox of reference date).

For the case of a spherical spacecraft, the drag acceleration is computed simply using the general form of Equation (1.1) and

$$\ddot{\vec{R}}_D = -S_s \rho\ \vec{V}_{rel} \left|\vec{V}_{rel}\right| \tag{1.3}$$

where

$$S_s = \frac{1}{2} C_D \left(\frac{A}{m}\right) = \frac{1}{2}\left(\frac{\pi\ d^2}{4m}\right) \tag{1.4}$$

where d is the spacecraft diameter, and m is the mass. If there is propulsive thrust acting, the mass is variable and is represented as a polynomial in the burn time. The polynomial coefficients are assumed to be known inputs.

When the spacecraft configuration is more complicated than a sphere, it is necessary to know the attitude, in addition to the orbit, in order to model the aerodynamic force.

It is not necessary to compute the entire direction cosine matrix (from body to inertial axes) when the spacecraft is a cylinder (with enclosing end plates). Due to the axial symmetry, it is only necessary to know the direction cosines of the cylinder axis.

The unit vector

$$\hat{X}_B = q_{11}\hat{i} + q_{21}\hat{j} + q_{31}\hat{k} \tag{1.5}$$

then gives the axis orientation in the inertial coordinate frame. The force component along the axis is proportional to the square of the velocity component normal to the end plates. The normal force component is proportional to the square of the velocity component normal to the cylinder. Therefore, the velocity relative to the atmosphere is resolved into normal and axial components in order to obtain the total acceleration for the cylindrical spacecraft as

$$\overline{N} = S_c \hat{X}_B \times \left(\hat{X}_B \times \overline{V}_{rel}\right)\left|\hat{X}_B \times \overline{V}_{rel}\right| \quad \overline{A} = -S_e \hat{X}_B \left(\hat{X}_B \cdot \overline{V}_{rel}\right)\left|\hat{X}_B \cdot \overline{V}_{rel}\right| \quad \ddot{\overline{R}}_D = \rho\left(\overline{N} + \overline{A}\right) \tag{1.6}$$

In these equations

$$S_c = \frac{1}{2}\left(\frac{C_{N_C}}{\sin^2 a}\right)\left(\frac{A}{m}\right) = \frac{1}{2}\left(\frac{4}{3}\right)\left(\frac{LD}{m}\right) = \frac{2LD}{3m}$$
$$S_e = \frac{1}{2}\left(\frac{C_{N_P}}{\cos^2 a}\right)\left(\frac{A}{m}\right) = \frac{1}{2}(2)\left(\frac{\pi\ d^2}{4m}\right) = \frac{\pi\ d^2}{4m} \tag{1.7}$$

where is the length of the cylinder and d is the diameter. As before, m is the spacecraft mass, which may be variable.

A third type of simple satellite configuration is a cylinder with solar paddles, mounted on pivots which are orthogonal to the cylinder axis. The incidence angle defines the angle between the axis and the paddle surface. The spacecraft axis system is chosen so the x-axis corres-ponds with the cylinder axis, is the pivot axis, and z is orthogonal to and . The y axis is directed so that positive corresponds with positive rotation about y, according to the right-hand rule.

This configuration is not axisymmetric and therefore requires the calculation of the complete transformation matrix (from body to inertial axes). It is most convenient to transform the relative wind velocity into spacecraft body axes, compute the force components in this frame, and then transform the result back into the inertial coordinate frame. This leads to the following equations for the aerodynamic acceleration:

$$\bar{V}_B = Q^{-1}V_{rel} = \dot{x}_B\bar{i}_B + \dot{y}_B\bar{j}_B + \dot{z}_B\bar{k}_B \quad V_N = \dot{x}_B \sin i_p + \dot{z}_B \cos i_p \quad F_{x_B} = -S_e\dot{x}_B|\dot{x}_B| - S_p V_N |V_N| \sin i_p$$

$$F_{y_B} = -S_c\dot{y}_B\sqrt{\dot{y}_B^2+\dot{z}_B^2} \quad F_{z_B} = -S_c\dot{z}_B\sqrt{\dot{y}_B^2+\dot{z}_B^2} - S_p V_N|V_N|\cos i_p \quad \ddot{\bar{R}}_D = -\rho Q\bar{F}_B \tag{1.8}$$

The definitions of S_c and S_e are the same as in Equations (1.7). The solar paddle contribution is

$$S_p = \frac{1}{2}\left(\frac{C_{N_p}}{\cos^2 a}\right)\left(\frac{A_p}{m}\right) = \frac{1}{2}(2)\left(\frac{A_p}{m}\right) = \frac{A_p}{m} \tag{1.9}$$

where the paddle area A_p is an input constant.

The representation of the aerodynamic forces in Equations (1.9) does not con-sider the effect of mutual shadowing or shielding from the free-stream flow between the cylindrical and solar paddle surfaces. Such effects are geometri-cally very complex, particularly if multiple interference reflections between cylinder and paddles are considered. The simplifications resulting from the neglect of this phenomenon in Equations (1.8) are thought to be consistent with the original assumption of purely specular reflection in the specification of the individual surface type coefficients.

The factor ρ in the three expressions for $\ddot{\bar{R}}_D$ is not simply the atmospheric density . It also includes a scale factor

$$\rho = \rho_a\left(1+\rho_1\right) \tag{1.10}$$

to permit an adjustment of the ρC_F product. A default value of $\rho_1 = 0$ is set in the program. However, this value can be modified by user input, or it can be estimated in the differential correction process. Adjustment of ρ_1 does not alter the instantaneous direction of $\ddot{\bar{R}}_D$; it simply changes the magnitude.

Jacchia-Roberts Atmospheric Model

Jacchia (1963) defined two empirical profiles to represent temperature as a function of altitude and exospheric temperature. One profile is defined for the altitude range from 90 to 125 km and the other for the region above 125 km. Jacchia used these temperature functions in the appropriate thermodynamic differential equations to determine density as a function of altitude and exospheric temperature. He assumed that mixing is predominant between 90 and 100 km, and substituted the low alti-tude temperature profile into the barometric differential equation for this regime. Diffusive equilibrium was assumed above 100 km, leading to the use of the low altitude temperature profile in the diffusion differential equation for altitudes between 100 and 125 km and the high altitude temperature profile for altitudes above 125 km.

Jacchia solved these differential equations by integrating them numerically over the altitude regions for various constant values of exospheric temperature, as-suming fixed boundary conditions at the 90 km lower altitude limit. He then tabulated these numerical results for use in the simulation of aerodynamic drag effects upon satellites. Most mechanizations of this model atmosphere in com-puter programs have involved some means for storing the tabular data and for interpolating values at altitudes computed by the trajectory integration and at exospheric temperatures calculated by the Jacchia formulas. Although the densities determined by this model are accurate, these mechanizations are gen-erally slow running and/or require large blocks of core storage. In addition, the absence of explicit analytic expressions means that the drag partial deriva-tives must be calculated numerically. Roberts (1971) presented a method for evaluating the Jacchia (1971) model analyti-cally and this formulation is commonly used in orbit determination systems. Roberts found that the barometric and diffusion differential equations could be integrated by partial fractions, using Jacchia's low altitude temperature profile for the range from 90 to 125 km. Above 125 km, Roberts used a different as-ymptotic function than the one introduced empirically by Jacchia in order to obtain an integrable form. Apart from difficulties of numerical computations with finite numbers of digits, the Roberts analytic expressions match the Jacchia results exactly from 90 to 125 km and to a close approximation above 125 km. The existence of these analytic expressions makes possible the computation of analytic forms for the drag partial derivatives. Since the Roberts formulas were derived for the Jacchia 1970 model, his constants have been adjusted for the later 1971 model. In addition, an error has been corrected in the function given by Roberts (1971) in Equations (12).

The computations begin with equations given in the Jacchia report to determine the exospheric temperature and corrections to the standard density due to various effects.

Before execution of a trajectory generation, an orbit determination system determines the total time span of interest. Then, from a permanent data file, one set of values of geomagnetic activity data and two sets of solar flux data are retrieved. The geomagnetic data set is the 3-hour geomagnetic planetary index K_p. One set of the solar flux data is the daily average 10.7 cm. solar flux, $F_{10.7}$, as observed at an appropriate solar observ-atory at high latitudes; the other set is the 81-day running average (centered at the day of interest), $\bar{F}_{10.7}$, of $F_{10.7}$. The solar flux data are substituted into the equation

$$T_c = 379^0 + 3.^0 24\bar{F}_{10.7} + 1^0_. 3\left[F_{10.7} - \bar{F}_{10.7}\right] \quad (2.1)$$

for determining the nighttime minimum global exospheric temperature for zero geomagnetic activity. The preprocessing computation of Equation (2.1) is done for each day of the time span of interest, beginning one day prior to the start of the trajectory. The daily values of T_c and the 3-hourly values of K_p (beginning $6^h_. 7$ prior to trajectory start) are stored in a working file for use in the com-putation of the trajectory.

At each trajectory integration time point, the value of T_c is retrieved from the working file for the day before the current time. This accounts for the fact that there is a one-day lag in the temperature variation with respect to solar flux change. This value of T_c is used to compute the uncorrected exospheric tem-perature T_1 from the formula

$$T_1 = T_c\left\{1 + 0.3\left[\sin^{2.2}\theta + \left(\cos^{2.2}\eta - \sin^{2.2}\theta\right)\cos^{3.0}\frac{\tau}{2}\right]\right\} \quad (2.2)$$

where

$$\eta = \frac{1}{2}\left|\phi - \delta_s\right| \quad \theta = \frac{1}{2}\left|\phi + \delta_s\right| \quad \tau = H - 37^0_. 0 + 6^0 0\sin\left(H + 43^0_. 0\right) \quad (-\pi < \tau < \pi)$$

δ_s is the sun’s declination, and

$$\phi = \tan^{-1}\left\{\frac{1}{(1-f)^2}\left[\frac{X_3}{\left(X_1^2 + X_2^2\right)^{1/2}}\right]\right\} \quad (2.3)$$

is the geodetic latitude.

The constant f is the geodetic flattening and X_1, X_2, X_3 are the components of the unit position vector of the spacecraft in true of date coordinates. The parameter

$$H = 180^0 0\left\{\frac{(S_1X_2 - S_2X_1)}{\pi\left|S_1X_2 - S_2X_1\right|}\cos^{-1}\left[\frac{S_1X_1 + S_2X_2}{\left(S_1^2 + S_2^2\right)^{1/2}\left(X_1^2 + X_2^2\right)^{1/2}}\right]\right\} \quad (2.4)$$

is the local hour angle of the sun (counted from upper culmination). The components S_1, S_2, S_3 comprise the unit vector to the sun in true of date coordinates.

The effect of geomagnetic activity upon atmospheric temperature and density shows a lag behind the geomagnetic disturbance. Thus, the value of K_p is retrieved from the working file for a time $6^h_. 7$ earlier than the current integration time point. The correction to exospheric temperature is given by

$$\left.\Delta T_\infty = 28^0_. 0k_p + 0^0_. 03e^{k_p} \quad (Z \geq 200\ km),\ \Delta T_\infty = 14^0_. 0k_p + 0^0_. 02e^{k_p} \quad (Z < 200\ km)\right\} \quad (2.5)$$

The corrected exospheric temperature is

$$T_\infty = T_1 + \Delta T_\infty \quad (2.6)$$

and the inflection point temperature is

$$T_x = 371^0_. 6678 + 0.0518806T_\infty - 294^0_. 3505e^{-0.00216222\,T_\infty} \quad (2.7)$$

These two temperatures together with the spacecraft altitude, are used in the Roberts equations to compute the standard density value. However, a number of corrections must be applied to the standard density values in order to account for various physical effects. These corrections are given by formulas from Jacchia (1971), and will be presented before proceeding to the Roberts equations.

In addition to the correction to the exospheric temperature, there is another direct geomagnetic effect on the standard density below 200 km

$$\left(\Delta\log_{10}\rho\right)_G = 0.012k_p + 1.2\times10^{-5}e^{k_p} \quad (2.8)$$

The semi-annual density variation is given by the following relationships (for altitude Z in km):

$$(\Delta \log_{10} \rho)_{SA} = f(Z) g(t) \qquad (2.9)$$

where

$$f(Z) = \left(5.876 \times 10^{-7} Z^{2.331} + 0.06328\right) e^{-.002868Z}$$

$$g(t) = 0.02835 + \left[0.3817 + 0.17829 \sin\left(2\pi\ \tau_{SA} + 4.137\right)\right]$$

$$\times \sin\left(4\pi\ \tau_{SA} + 4.259\right)$$

$$\tau_{SA} = \Phi + 0.09544\left\{\left[\frac{1}{2} + \frac{1}{2}\sin(2\pi\Phi + 6.035)\right]^{1.65} - \frac{1}{2}\right\} \qquad \Phi = \frac{JD_{1958}}{365.2422}$$

In the last equation JD_{1958} is the number of Julian Days from January 1, 1958.

The correction for the seasonal latitudinal variation of the lower thermosphere is

$$(\Delta \log_{10} \rho)_{LT} = 0.014(Z - 90) e^{\left[-0.0013(Z-90)^2\right]} \times \qquad (2.10)$$

$$\times \sin\left(2\pi\Phi + 1.72\right) \sin\phi \left|\sin\phi\right| \qquad (2.11)$$

Finally, the correction for the seasonal latitudinal variation of helium is

$$(\Delta \log_{10} \rho)_{He} = 0.65\left|\frac{\delta_s}{\varepsilon}\right| \left[\sin^3\left(\frac{\pi}{4} - \frac{\phi\delta_s}{2|\delta_s|}\right) - 0.35355\right] \qquad (2.12)$$

where ε is the obliquity of the ecliptic.

As mentioned earlier, for altitudes below 125 km Roberts used the same tem-perature profile that Jacchia used, i.e.,

$$T(Z) = T_x + \frac{d_1}{35^4} \sum_{n=0}^{4} C_n Z^n \qquad (2.13)$$

where

$$d_1 = T_x - T_0 \quad T_0 = 183^{0}0K \quad C_0 = -89284375.0 \quad C_1 = 3542400.0\ km^{-1}$$

$$C_2 = -52687.5\ km^{-2} \qquad C_3 = 340.5\ km^{-3} \quad C_4 = -0.8\ km^{-4} \qquad (2.14)$$

and where T_x is the inflection point temperature (at $Z_x = 125$ km) given by Equation (2.7). Roberts substituted the temperature profile, given by Equation (2.14), in the barometric differential equation and integrated by partial fractions to obtain

$$\rho_s(Z) = \left(\frac{\rho_0 T_0}{M_0}\right) \frac{M(Z)}{T(Z)} F_1^k \exp(kF_2) \qquad (2.15)$$

as the expression for density for $90 < Z \leq 100$ km, where the subscript "0" refers to conditions at 90 km.

The mean molecular weight is given as

$$M(Z) = \sum_{n=0}^{6} A_n Z^n \qquad (2.16)$$

where

$A_0 = -435093.363387$
$A_1 = 28275.5646391\ km^{-1}$
$A_2 = -765.33466108\ km^{-2}$
$A_3 = 11.043387545\ km^{-3}$
$A_4 = -0.08958790995\ km^{-4}$
$A_5 = 0.00038737586\ km^{-5}$
$A_6 = -0.000000697444\ km^{-6}$

These constants give a value of $M(90) = M_0 = 28.82678$, which is not too different from the sea-level mean molecular mass M_s of 28.960.

The value of density at the lower limit is assumed to be constant at $\rho_0 = 3.46 \times 10^{-9}\ gm/cm^3$.

The constant k in Equation (2.15) is $k = -35^4 g_s R_a^2 / R d_1 C_4$

where $g_s = 9.80665 m/\sec^2$ = sea level acceleration due to gravity, $R_a = 6356.766\ km$ = mean equatorial radius, $R = 8.31432\ Joules/{}^0K$ = mole (universal gas constant)

The functions F_1, F_2 in Equation (2.15) are

$$F_1 = \left(\frac{Z + R_a}{90 + R_a}\right)^{P_1} \left(\frac{Z - r_1}{90 - r_1}\right)^{P_2} \left(\frac{Z - r_2}{90 - r_2}\right)^{P_3} \left(\frac{Z^2 - 2XZ + X^2 + Y^2}{8100 - 180X + X^2 + Y^2}\right) \qquad (2.17)$$

$$F_2 = (Z - 90)\left[A_6 + \frac{P_5}{(Z + R_a)(90 + R_a)}\right] + \frac{P_6}{Y}\tan^{-1}\left[\frac{Y(Z - 90)}{Y^2 + (Z - X)(90 - X)}\right]$$

In these functions r_1 and r_2 are the two real roots and X and Y are the real and imaginary parts $Y > 0$, respectively, of the complex conjugate roots of the quadratic form

$$P(Z) = \sum_{n=0}^{4} C_n^* Z^n \qquad (2.18)$$

with coefficients

$$C_0^* = \frac{35^4 T_x}{C_4 d_1} + \frac{C_0}{C_4} \quad C_n^* = \frac{C_n}{C_4} \qquad 1 \le n \le 4$$

for values of C_n given by Equations (2.14). The parameters p_i in the functions F_i are

$$p_2 = \frac{S(r_1)}{U(r_1)} \quad p_3 = \frac{-S(r_2)}{U(r_2)} \quad p_5 = \frac{S(-R_a)}{V}$$

$$p_4 = \left\{B_0 - r_1 r_2 R_a^2 \left[B_4 + B_5(2X + r_1 + r_2 - R_a)\right] + W(r_1)p_2 - r_1 r_2 B_5 R_a (X^2 + Y^2) + W(r_2)p_3 \;\; + r_1 r_2 (R_a^2 - X^2 - Y^2)p_5\right\} X^*$$

$$p_6 = B_4 + B_5(2X + r_1 + r_2 - R_a) - p_5 - 2(X + R_a)p_4 \;\; - (r_2 + R_a)p_3 - (r_1 + R_a)p_2$$

$$p_1 = B_5 - 2p_4 - p_3 - p_2$$

In these parameters

$$X^* = -2 r_1 r_2 R_a (R_a^2 + 2XR_a + X^2 + Y^2) \quad V = (R_a + r_1)(R_a + r_2)(R_a^2 + 2XR_a + X^2 + Y^2) \tag{2.19}$$

$$U(r_i) = (r_i + R_a)^2 (r_i^2 - 2Xr_i + X^2 + Y^2)(r_1 - r_2) \quad W(r_i) = r_1 r_2 R_a (R_a + r_i)\left(R_a + \frac{X^2 + Y^2}{r_i}\right)$$

The function $W(r_i)$ is corrected from an erroneous expression given by Roberts (1971). Finally, the coefficients B_n and the function $S(Z)$ are given by

$$B_n = a_n + \beta_n \frac{T_x}{T_x - T_0} \qquad S(Z) = \sum_{n=0}^{5} B_n Z^n \qquad (n = 0,1,\ldots,5)$$

where , $a_0 = 3144902516.672729$,
$a_1 = -123774885.4832917$,
$a_2 = 1816141.096520398$,
$a_3 = -11403.31079489267$,
$a_4 = 24.36498612105595$,
$a_5 = 0.008957502869707995$ and
$\beta_0 = -52864482.17910969$,
$\beta_1 = -16632.50847336828$,
$\beta_2 = -1.308252378125$, $\beta_3 = 0.0$, $\beta_4 = 0.0$,
$\beta_5 = 0.0$

As noted above, Equation (2.15) is valid below $Z = 100$ km, where mixing is assumed to be predominant. How ever, diffusive equilibrium is assumed above $Z = 100$ km; hence, the profile given by Equation (2.13) was substituted into the diffusion differential equations (one for each constituent of the atmosphere) and integrated by partial fractions by Roberts to yield for $100 < Z \le 125$ km

$$\rho_s(Z) = \sum_{i=1}^{5} \rho_i(Z) \tag{2.20}$$

Rigorously, the density at 100 km, $\rho(100)$, should be evaluated by means of Equation (2.15) for the particular exospheric temperature T_∞ of interest. However, since the evaluation of that equation is computationally expensive, it is preferable to avoid adding that expense to that already necessary to compute Equation (2.20). This is avoided by precomputing values of $\rho(100)$, using Equation (2.15), for a series of values of T_∞. These values have been least-squares curve fitted by the polynomial

$$\frac{\rho(100)}{M_s} = \sum_{n=0}^{6} \zeta_n T_\infty^n \tag{2.21}$$

where
$\zeta_0 = 0.1985549 \times 10^{-10}$
$\zeta_1 = -0.183349 \times 10^{-14}$
$\zeta_2 = 0.1711735 \times 10^{-17}$
$\zeta_3 = -0.1021474 \times 10^{-20}$
$\zeta_4 = 0.3727894 \times 10^{-24}$
$\zeta_5 = -0.7734110 \times 10^{-28}$
$\zeta_6 = 0.7026942 \times 10^{-32}$
$M_s =$ the sea level mean molecular mass = 28.96 gm/mole.

This approximation is used in Equation (2.20).

The constituent mass densities for altitudes between 100 and 125 km are given by

$$\rho_i(Z) = \rho(100)\frac{M_i}{M_s}\mu_i \left[\frac{T(100)}{T(Z)}\right]^{1+a_i} F_3^{M_i k} \exp(M_i k F_4) \tag{2.22}$$

The identification of the constituents and the values of the corresponding constants in Equation (2.22) are given in Table 2.1.

Table 2.1 - Atmospheric Constituents and Related Constants

Index i	Constituent	Molecular mass M_i (grams / mole)	Thermal diffusion coefficient a_i	μ_i, constituent number density $\times (M_s / \rho(100))$ divided by Avogadro's number
1	N_2	28.0134	0	0.78110
2	Ar	39.948	0	0.93432×10^{-2}
3	He	4.0026	-0.38	0.61471×10^{-5}
4	O_2	31.9988	0	0.161778
5	O	15.9994	0	0.95544×10^{-1}
6	H	1.00797	0	--

Hydrogen is an insignificant constituent at altitudes below 125 km; hence, it is not included in Equations (2.20) and (2.22). The temperature at 100 km is given by Equation (2.13) in the form

$$T(100) = T_x + \Omega d_1 \tag{2.23}$$

where $\Omega = 35^{-4}\sum_{n=0}^{4} C_n(100)^n = -0.94585589$

is the pre-computed value of the polynomial for 100 km. The parameter k in Equation (2.22) is the same as defined previously, and the functions F_3 and F_4 are given as

$$F_3 = \left(\frac{Z+R_a}{R_a+100}\right)^{q_1}\left(\frac{Z-r_1}{100-r_1}\right)^{q_2}\left(\frac{Z-r_2}{100-r_2}\right)^{q_3}\left(\frac{Z^2-2XZ+X^2+Y^2}{100^2-200X+X^2+Y^2}\right)^{q_4}$$

$$F_4 = \frac{q_5(Z-100)}{(Z+R_a)(R_a+100)} + \frac{q_6}{Y}\tan^{-1}\left[\frac{Y(Z-100)}{Y^2+(Z-X)(100-X)}\right] \quad (2.24)$$

The parameters q_i are defined as

$$q_2 = \frac{1}{U(r_1)} \quad q_3 = \frac{-1}{U(r_2)} \quad q_5 = \frac{1}{V} \quad q_4 = \left\{+r_1r_2\left(R_a^2-X^2-Y^2\right)q_5 + W(r_1)q_2 + W(r_2)q_3\right\}X^*$$

$$q_6 = -q_5 - 2(X+R_a)q_4 - (r_2+R_a)q_3 - (r_1+R_a)q_2 \quad q_1 = -2q_4 - q_3 - q_2$$

and $X, Y, r_1, r_2, X^*, V, U(v)$, and $W(v)$ are the same as defined previously.

Finally, diffusive equilibrium is still assumed for the region above 125 km, but the temperature profile given by Equation (2.13) is no longer valid. Jacchia defined the temperature for the upper region by the empirical asymptotic function

$(i = 1,2,......,5)$ Z_x γ_6

$$T(Z) = T_x + \frac{2}{\pi}(T_\infty - T_x)\tan^{-1}\left\{0.95\pi\left(\frac{T_x-T_0}{T_\infty-T_x}\right)\left(\frac{Z-125}{35}\right)\left[1+4.5\times10^{-6}(Z-125)^{2.5}\right]\right\} \quad (2.25)$$

In order to be able to integrate the diffusion differential equations in closed form, Roberts replaced Jacchia's Equation (2.25) with the function

$$T(Z) = T_\infty - (T_\infty - T_x)\exp\left[-\left(\frac{T_x-T_0}{T_\infty-T_x}\right)\left(\frac{Z-125}{35}\right)\left(\frac{\ell}{R_a+Z}\right)\right] \quad (2.26)$$

This temperature profile is continuous at $Z_x = 125$ km regardless of the choice of the parameter ℓ. The slope is continuous at if

$$\ell = 1.9(R_a + Z_x) = 12315.3554 \ km$$

The value of ℓ is computed by a procedure to be described later.

Integration of the diffusion differential equations for the temperature profile given by Equation (2.26) yields, for the first five constituents in Table 2.1

$$\rho_i(Z) = \rho_i(125)\left(\frac{T_x}{T}\right)^{1+\alpha_i+\gamma_i}\left(\frac{T_\infty-T}{T_\infty-T_x}\right)^{\gamma_i} \quad (2.27)$$

where

$$\gamma_i = \frac{M_i g_0 R_a^2}{R\ell T_\infty}\left(\frac{T_\infty-T_x}{T_x-T_0}\right)\left(\frac{35}{6481.766}\right) \quad (2.28)$$

The constituent mass densities at 125 km can be obtained rigorously from Equation (2.22). However, as in the case of the density at 100 km, a curve-fitting approximation is made to give (for

$$i = 1,2,.......,5): \ \log_{10} d_i(125) = \sum_{j=0}^{6}\delta_{ij}T_\infty^j \quad (2.29)$$

as a function of exospheric temperature, where d_i is the constituent number density divided by Avogadro's number $(\rho_i = M_i d_i)$. The polynomial coefficients δ_{ij} in Equation (2.29) have been determined for best fits to the values corre-sponding to Equation (2.22), and are given in Table 2.2.

The value of the helium density computed by Equation (2.27) must be corrected for the seasonal latitudinal variation as given by Equation (2.12). The specific form is

$$[\rho_3(Z)]_{corrected} = \rho_3(Z)10^{(\Delta\log_{10}\rho)He}$$

Above 500 km the concentration of hydrogen ($i = 6$ in Table 2.1) becomes suf-ficiently large that it also must be taken into account

$$\rho_6(Z) = \rho_6(500)\left[\frac{T(500)}{T(Z)}\right]^{(1+\alpha_6+\gamma_6)}\left[\frac{T_\infty-T(Z)}{T_\infty-T(500)}\right]^{\gamma_6} \quad (2.30)$$

where the hydrogen density at 500 Km is

$$\rho_6(500) = \frac{M_6}{A}10^{[73.13-(39.4-5.5\log_{10}T_{500})\log_{10}T_{500}]} \quad (2.31)$$

For exospheric temperatures lower than approximately $600^0 K$, the relative concentration of hydrogen is significant at altitudes lower than 500 Km; however, the resulting density error is partially compensated for by the least squares fitting of Roberts' parameter ℓ (Equation 2.35)

In Equation (2.30), is computed by means of Equation (2.28). The quantity A in Equation (2.31) is Avogadro's number $\left(A = 6.02257\times10^{23}\right)$. The temperature at 500 Km is computed in Equation (2.26). Finally, the constituents are summed to yield

$$\rho_s(Z) = \sum_{i=1}^{6} \rho_i(Z) \qquad (2.32)$$

as the standard density for the region $Z > 125$ km.

The standard density, as computed by Equations (2.15), (2.20), or (2.32) must be corrected for geomagnetic activity (by Equation (2.8)), the semi-annual variation (by Equation (2.9)), and the seasonal latitudinal variation of the lower thermosphere (by Equation (2.11)). These effects are summed logarithmically to obtain the standard density.

The standard densities, as computed by Equations (2.15) and (2.20) for the region $90 < Z \le 125$ km, agree exactly with values published by Jacchia (1971).

Above 125 km, however, the values given by Equation (2.32) do not agree exactly with the Jacchia data, due to Roberts' introduction of a differ-ent form (Equation 2.26) for the temperature profile at the higher altitudes. Values of the parameter ℓ in Roberts' temperature profile were determined for a series of exosphere temperatures, such that the resulting density profiles versus altitude (from 125 km to 2500 km) gave the best least squares fit to the Jacchia tabulated data. Three sample fits are shown in Figure 4.3 for low, medium, and high values of the exosphere temperature. Note that the maximum deviation from the Jacchia values is less than 6.7%. The best-fit values of ℓ are given by the interpolation polynomial as a function of exosphere temperature according to Eq. (2.35)

$$\ell = \sum_{j=0}^{4} \ell_j T_\infty^j \qquad (2.35)$$

with coefficients

$\ell_0 = 0.1031445 \times 10^5$, $\ell_1 = 0.2341230 \times 10^1$, $\ell_2 = 0.1579202 \times 10^{-2}$, $\ell_3 = -0.1252487 \times 10^{-5}$, $\ell_4 = 0.2462708 \times 10^{-9}$

$$(\Delta \log_{10} \rho)_{corr} = (\Delta \log_{10} \rho)_G + (\Delta \log_{10} \rho)_{SA} + (\Delta \log_{10} \rho)_{LT} \qquad (2.33)$$

Thus, the final corrected density is

$$\rho(Z) = \rho_s(Z) 10^{(\Delta \log_{10} \rho)_{corr}} \qquad (2.34)$$

computed to best fit the optimum ℓ values. Equation (2.35) should be programmed in an orbit determination system to provide the means for selecting ℓ in Equation (2.26). In general, the values of ℓ are such that the slope of the temperature profile is discontinuous at km, but this is not thought to be of any serious consequence.

Table 2.2 - Polynomial Coefficients for Constituent Densities at 125 km.

Degree of polynomial term (j)	Constituent (i) (1) N_2	(2) Ar	(3) He	(4) O_2	(5)
0	0.1093155×10	0049405×10^1	0.7646886×10^1	0.9924237×10^1	0.1097083×1…
1	0.1186783×10	02382822×10^{-2}	$-0.4383486 \times 10^{-3}$	0.1600311×10^{-2}	0.6118742×1…
2	-0.1677341×10	03391366×10^{-5}	0.4694319×10^{-6}	-2274761×10^{-5}	$-0.1165003 \times$…
3	0.1420228×10	02909714×10^{-8}	$-0.2894886 \times 10^{-9}$	0.1938454×10^{-8}	0.9239354×1…
4	-0.7139785×10	01481702×10^{-11}	$0.9451989 \times 10^{-13}$	$-0.9782183 \times 10^{-12}$	$-0.3490739 \times$…
5	0.1969715×10	04127600×10^{-15}	$-0.1270838 \times 10^{-16}$	$0.2698450 \times 10^{-15}$	0.5116298×1…
6	-0.2296182×10	04837461×10^{-19}	0.0	$-0.3131808 \times 10^{-19}$	0.0

Final Remarks

Although several modifications and alternatives models have been proposed by several authors, the Jacchia Atmospheric Model (Jacchia, 1960, 1963, 1964, 1971, 1977), as developed in full details by Roberts (1971) still remains the most adopted model for application to the motion of an artificial satellite of the Earth. This can be seen in recent works by Der and Bonavito (1998), Montenbruck and Gill (2000), Valiado (2004), Beutler (2005) among others. The Harris-Priester (!962, 1965) simplified model was not successful in representing correctly the atmosphere density for most usual altitudes. First reliable information was made available in the US Standard Atmosphere (1976). Several versions of the COSPAR International Reference Atmosphere (CIRA) were developed and made available in 1972, 1986 and 1992, all of these essentially based on Jacchia´s original model. As far the identification of the data underlying the Jacchia Atmospheric Model and other models, the most com-plete information still remain the MSISe-90 (Mass Spectrometry and Incoherent Scatter) published by NASA Goddard Space Flight Center (Hedin, 1991). We finalize this review by observing that, actually, no model is completely exact to represent the atmosphere density at any height and this topic certainly represents one of the most difficult problems in the orbit prediction prediction of an artificial satellite of the Earth

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