EXPERIMENTAL DAMPING IDENTIFICATION OF A CANTILEVERED BEAM

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Abstract. *: This work addresses the modeling and experimental damping of a cantilevered beam. Its objectives are to describe the design and the implementation of an experiment to study the dynamics and the experimental identification of the beam's damping matrix. To do so an experimental setup was assembled composed by a cantilevered beam a impact hammer and a accelerometer. The dynamic modeling and an experimental modal analysis identification in the frequency domain was applied to obtain the vibrations modes of the beam and a procedure to the identification of the system damping matrix based on Genetic Algorithms method's was used do complete the model. The results are quite good and show a good agreement between the real and experimental systems.*

Keywords: *flexible structure, damping matrix, identification, genetic algorithms*

1. INTRODUCTION

The field of damping matrix identification is one that still has some issues to be answered by the engineering community. The damp's effects are very well known but its characterization is a problem not solved yet. Pilkey, Roe and Iman (1997) propose two damping identification methods, one interactive and another that uses least square. Adhikari (2002) proposes a method, based on the poles and residues of the measured transfer functions, associated with Lancaster's method. Also investigations on dynamic modeling and control of flexible structure have attracted a great deal of interest due to special applications in the control of large flexible manipulator systems, with large work-space and simultaneous requirements of great precision at its end-effector's positioning, Lew, and Trudnowski, (1996). In these applications the active vibration control of the macro flexible manipulator requires considerable control energy. To address this subject an experimental setup was assembled at the Linear and Non-Linear Vibrations Laboratory in the Mechanical Engineering Department of Taubate's University (UNITAU), and consists of a cantilevered beam, a accelerometer and a impact hammer. The flexible beam has a low stiffness in the vertical direction.

In this work we describe the analytical modeling together with model validation studies carried out through experimental modal testing and parametric system identification studies in the frequency domain. An impulse (applied by the impact hammer) excites the beam and the signals provided by the hammer and the accelerometer, located on the tip of the beam, is acquired for further analysis. A Digital Signal Analyzer HP-35660A, used in data acquisition and results analysis, completes the experimental setup. Preliminaries results show a good agreement between experimental and analytical models.

2. DYNAMIC MODELING OF THE SYSTEM

The generalized Lagrangean approach and the Assumed Mode Method are used to derive the analytical model of the cantilevered beam structure. The kinetic energy of the system can be expressed as, Soares et al (1997):

$$T(t) = \int_o^L \rho(x)\left[\dot{y}(x,t)^2\right]dx \tag{1}$$

The potential energy of the distributed parameter system does not take in account the shear deformation and the rotary inertia of the beam and is given by the following expression:

$$V(t) = \int_0^L EI\left[\frac{\partial^2 y(x,t)}{\partial x^2}\right]^2 dx \tag{2}$$

The discrete model of the system is obtained by *Ritz's Assumed Modes Method.* In this method the elastic displacement of the beam can be described as:

$$y(x,t) = \sum_{i=1}^{N} \phi_i(x)\eta_i(t) \tag{3}$$

Upon substitution on Eq. (1) and Eq. (2) above, gives, Junkins and Kim, (1993):

$$T(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\dot{\eta}_i(t)\dot{\eta}_j(t)\left[\int_0^L \rho(x)\phi_i(x)\phi_j(x)dx\right] \tag{4}$$

$$V(t) = \frac{1}{2}\int_0^L EI(x)\left[y''(x,t)\right]^2 dx \tag{5}$$

The comparison function, f *i(x)*, used in the numerical calculations, are chosen as admissible functions, Junkins and Kim, (1993):

$$\phi_i(x) = 1 - \cos\left(\frac{i\pi x}{L_b}\right) + \frac{1}{2}(-1)^{i+1}\left(\frac{i\pi x}{L_b}\right)^2 \tag{6}$$

Using the Lagrange equation, we can derive the equations of motion, as follow:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right] - \frac{\partial L}{\partial q_i} + \frac{\partial V}{\partial q_i} = F_i \quad \text{i= 1, 2, ...n} \tag{7}$$

The generalized coordinate vector, q , is given by:

$$\underline{q} = [\eta_1 \quad \eta_2 \quad \cdots \quad \eta_N]^T \tag{8}$$

Substituting eqs. (6) into (4) and (5), applying eq. (7), neglecting the coupled and second order terms, and since there are no external forces we can after some manipulation, derive the system equations in the mass (M) and stiffness (K) matrix form:

$$\underline{\underline{M}}\ddot{q} + \underline{\underline{K}}q = 0 \tag{9}$$

where M and K have the following form:

$$M_{ij} = \rho L \left\{ 1 + (-1)^{i+j} \left[\left({}^{i}\!/_{j} \right)^2 + \left({}^{j}\!/_{i} \right)^2 \right] + \frac{1}{2}\delta_{ij} + (-1)^{i+1}\left(\frac{i^2\pi^2}{6} \right) + (-1)^{j+1}\left(\frac{j^2\pi^2}{6} \right) + (-1)^{i+j}\left(\frac{i^2 j^2 \pi^4}{20} \right) \right\} \quad (10)$$

$$K_{ij} = EI\left[(-1)^{i+j} + \frac{1}{2}\delta_{ij} \right]\left(\frac{i^2 j^2 \pi^4}{L_b^3} \right) \quad (11)$$

Now it’s simple to get the state-space representation of the system in the form:

$$\underline{\dot{X}} = \underline{\underline{A}}\,\underline{X} + \underline{B}u \quad (12)$$

where the A e B matrix are:

$$\underline{\underline{A}} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{I}} \\ \underline{\underline{M}}^{-1}\underline{\underline{K}} & \underline{\underline{0}} \end{bmatrix} \quad (13)$$

$$\underline{B} = \begin{bmatrix} \underline{0} \\ \underline{\underline{M}}^{-1}\underline{F} \end{bmatrix} \quad (14)$$

3. THE ANALYTICAL TRANSFER FUNCTION

To obtain the analytical transfer functions, we used the physical parameters, listed in Tab. 1 below, for the multi-link flexible system.

Table 1 – Model parameter of the flexible beams

Aluminum density	ρ	2710	Kg/m³
Aluminum Young's modulus	E	7.1E10	N/m²
Beams width	eb	31.7 E-3	m
Beams height	hb	3.3E-3	m
Beams length	L_b	0.7	m
Beams cross-section area	A	10.46E-5	m^2
Beams moment of inertia	I	9.493E-11	m^4

Applying the Laplace transform in equation (12) with zero initial conditions, and using the model parameters listed in table 1, we can obtain the analytical transfer function. The Bode plots of the open loop system are obtained by substituting (s=jw) in the Laplace transfer functions shown below:

$$\underline{Y}(s) = \underline{\underline{C}}\left(s\underline{\underline{I}} - \underline{\underline{A}}\right)^{-1}.\underline{B}.U(s) \quad (15)$$

4. THE EXPERIMENTAL SETUP

In this work, an experimental setup, Fig.(1), was assembled at the Linear and Non-Linear Vibrations Laboratory in the Mechanical Engineering Department of Taubate’s University. The experiment was used to obtain the experimental data. The experimental setup includes a flexible aluminum beam, an impact hammer and a Digital Signal Analyzer HP-35660A.

Figure 1. The Experimental Setup

The experimental transfer functions were obtained by non-parametric system identification in the frequency domain. The flexible system was excited with impact hammer and the FFT of the output signal was obtained using a routine available in the Octave® software. The excitation was applied on the tip of the beam and the transfer function is shown in Fig. (2).

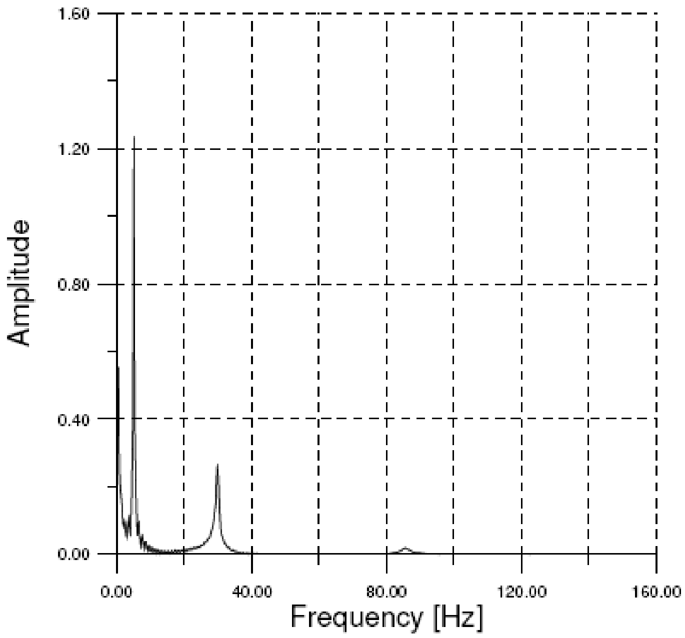

Figure 2. FFT of the accelerometer signal

5. MODEL VALIDATION AND PARAMETRIC IDENTIFICATION

Comparing the analytical and experimental frequencies, Tab. (2), we can observe some discrepancies between them. These errors are due, mainly, to sensor noise and unmodeled sensor dynamics. Nonetheless, the experimental transfer functions clearly show some sharpen peaks in the spectrum, which can be associated with the vibration modes of the flexible system. In order to check this assumption, a more accurate comparison can be done between the predicted and the experimentally determined modal frequencies. Table (2) below shows a comparison between the analytical and the experimental results, which suggests a good agreement between these analyses.

Table 2 - Comparison between analytical and experimental results

Mode nº	Analytical (Hz)	Experimental (Hz)
1º	5.1639344	5.5685326
2º	29.644809	34.897681
3º	84.726776	97.715208

6. IDENTIFICATION OF THE SYSTEM DAMPING MATRIX

In this paper, the problem to identification of the system damping matrix was writing like an optimization problem. We rewriting the analytical model by introducing the damping matrix unknown, define by Eq. (16).

$$\underline{\underline{A}} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{I}} \\ \underline{\underline{M}}^{-1}\underline{\underline{K}} & \underline{\underline{M}}^{-1}\underline{\underline{\hat{R}}} \end{bmatrix} \qquad (16)$$

And the optimization problem is to identify the system-damping matrix (R ˆ), which minimizes the error between the experimental, and the analytical response. We employ the Genetic algorithms (Michalewicz, Z., 1996) to solve this optimization problem.

Genetic algorithms are a popular form of evolutionary computation. In a few words, genetics algorithms operate on a population of artificial chromosomes by selectively reproducing the chromosomes of individuals with higher performance and applying random changes (mutation and crossover). Fitness Function is a performance criterion that evaluates the performance of each individual phenotype. Figure 3 illustrates this procedure, which is repeated for several generations.

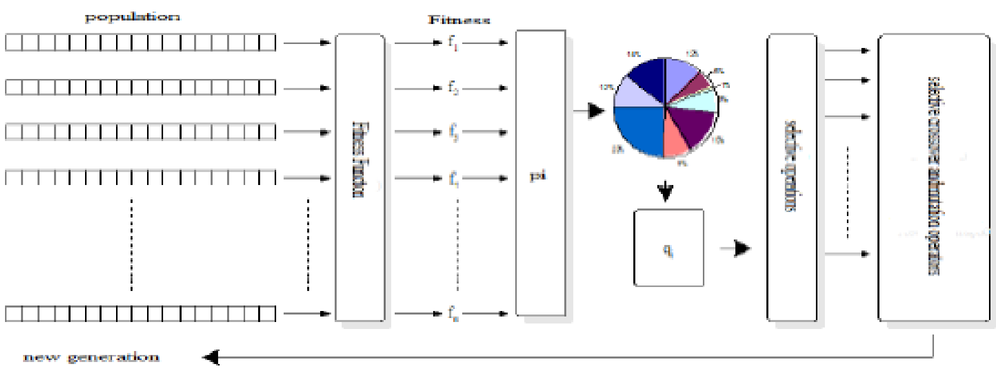

Figure 3. Genetic algorithms scheme

We employ a binary representation to the damping values of the system. Also, we set the population size to 50; the population mutation to 0.15; the population crossover to 0.45; and we consider 30 generations. The GA’s parameters are illustrated in Tab. (3).

Table 3 – GA parameters for the flexible structure

Population size	*p = 50*
Number of generations	*g = 30*
Initial mutation	*pc = 0.45*
Rate of mutation decrement	*mc =0.15*
Selection scheme	*roulette selection*
Crossover scheme	*randon crossover*
Variables Precision	*4 digits*
Variables range	*[-10,10]*

The quadratic square error was used to evaluate the performance of each individual phenotype, define in Eq. (17).

$$\Psi(x)=\frac{1}{N}\sqrt{(y-\hat{y})^T(y-\hat{y})} \tag{17}$$

Initially, an impulse was applied to the beam (with an impact hammer). The Fig. (4) illustrates the experimental impulse excitation of the cantilevered beam.

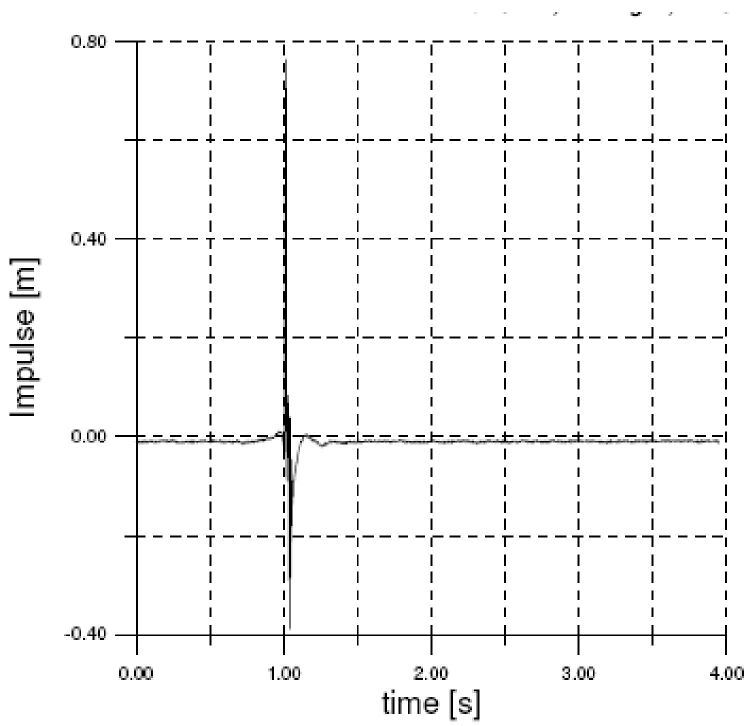

Figure 4. Input of the experimental setup

The output of the genetic algorithm leads to a damping matrix that completes the system modeling. The values of the damping matrix elements are shown in Eq. (18).

$$c=\begin{bmatrix}4.9582 & 4.1824 & 2.9015\\ 4.1824 & 4.0732 & 1.9945\\ 2.9015 & 1.9945 & 2.7482\end{bmatrix} \tag{18}$$

The Fig. (5) illustrates a comparison between the experimental (dotted curve) and the analytical responses (continuous curve).

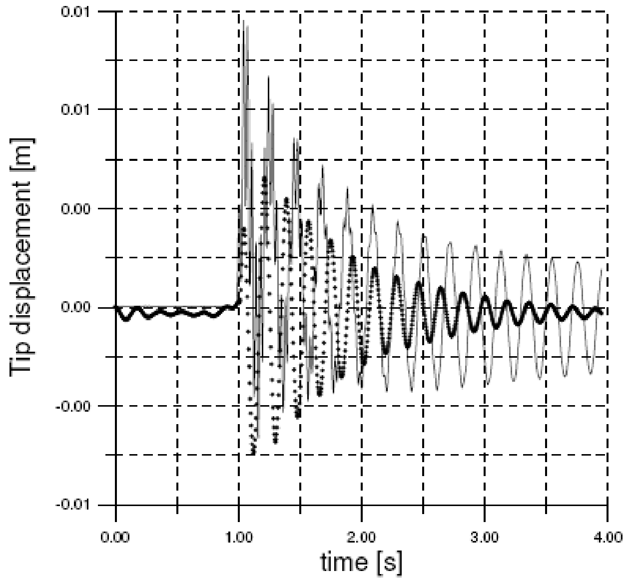

Figure 5. Comparison between the analytical and experimental responses

7. CONCLUDING AND REMARKS

The field of damping matrix identification is one that still has some issues to be answered by the engineering community. The damp's effects are very well known but its characterization is a problem not solved yet. Investigations on damping identification and dynamic modeling of flexible structure were addressed in this work. Its interest lies on the special applications in the modeling and control of large flexible manipulator systems, with large workspace and simultaneous requirements of great precision at its end-effector positioning. In these applications the active vibration control of the macro flexible manipulator requires considerable control energy. A genetic algorithm was implemented to the damping mass identification. The good agreement between both responses, Fig.(5), shows that the GAs are a quite good solution for this kind of problem. The work is in progress and the next steps will involve the control of a flexible structure using the identification data.

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