APPLICATION OF FUZZY OPTIMIZATION IN ENERGY SAVING

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ABSTRACT

In many power system problems, the use of optimization techniques has been important to reduce costs and losses of the system. One of the most important points in process is the computational cost to find the best solution. Sometimes, this cost may comprise the use of a technique to solve a problem. Number of constraints, number of variables and poor convergence speed are some examples of computational cost. This paper presents a development using fuzzy optimization process. This paper starts with theoretical aspects of the fuzzy optimization process and then, an example using power system energy saving is presented.

KEY-WORDS: Fuzzy optimization, optimization techniques, energy saving.

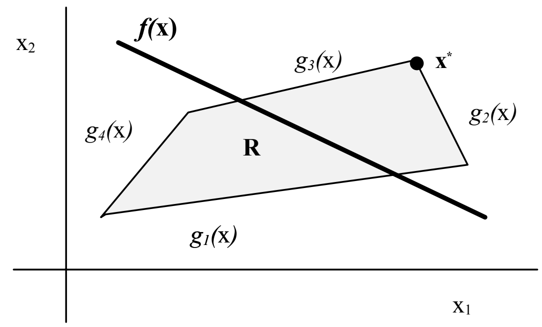
INITIAL CONSIDERATIONS

In the real world, it is not an easy task to find a solution of a given problem because many constraints and limitations must be taken into account during this process. Usually, only the most important constraints and limitations are chosen to be used during the solution search process. Another problem is that the solution can be not unique and it depends directly on the weight of each constraint. Hence, many optimization processes have been developed in the last decades to achieve the best solution in this search process. In addition, the computational problems are related to hardware processor speed, memory capacity and numerical techniques. However, the highly fast evolution of the computational world (hardware and software) allows optimization techniques that could not be used before to solve a specific problem, to be applied successfully now. Specifically for power system problems, decomposition techniques, partitioning techniques and parallel processing are examples of recent evolution of computational techniques.

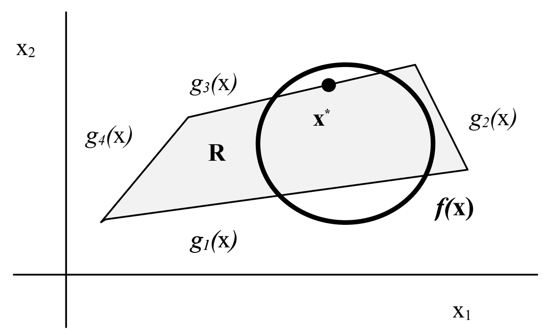
In many power system problems, the use of optimization techniques has been important to reduce costs and losses of the system. Unit commitment, economic dispatching, and optimal power flow are some areas where these techniques have been extensively used. For example, minimization of active power losses is one of the biggest challenges for power control operators. The achievement of this goal in real-time is a critical task. A possible solution for this problem is to use the Dantzig and Wolfe decomposition algorithm to partitioning the power system in many subsystems according to a geographic basis. The optimization process is applied to these subsystems, and the constraints are limited to local constraints and coupling bus constraints.

An optimization process can be defined as a maximization (or minimization) of an objective function, $f(x)$, subject to constraints of the problem, $g(x)$. These constraints define a feasible region **R**, i.e., a region that contains possible solutions of the problem. Two popular techniques have been developed for optimization process; they are linear programming and quadratic programming. Examples of these techniques, for two variables x_1 and x_2, are shown in Figure 1, where there are 4 linear inequality constraints, $g_i(x)$, that define the feasible region **R**, and the optimal solution is denoted by x^*.

It can be verified that in the linear programming the optimal solution occurs always at an extreme point (corner point, i.e., two active constraints) of the feasible region; while in the quadratic programming this solution can be located over only one of the constraints (one active constraint), i.e., this constraint is tangent to the objective function $f(x)$.



(a)



(b)

Fig.1 - Example of Techniques: (a) Linear Programming and (b) Quadratic Programming.

The two major drawbacks of these current optimization methods are: speed/convergence problems and correct representation of constraints. Usually, methods with fast speed present poor convergence, while slower methods have less convergence problems. In one hand, for example, Newton-Raphson (or other parallel tangent methods) presents a very good answer when the starting point is near the solution point; however, this method performance can be very dependent on the shape of the involved functions. On the other hand, bisection methods (e.g., Fibonacci, cubic, and quadratic searches) are slower than tangent methods but they are more reliable. Hybrid system schemes have been proposed. Initially, the procedure starts with a bisection method until the vicinity of the optimal point; then, the procedure changes the method to a parallel tangent method.

The second drawback, correct representation of constraints, is related to the difficulty to evaluate the correct value to be incorporated in the constraint equations. Sometimes, these constraints are not well defined by crisp functions, and the use of fuzzy values is recommended. Many fuzzy optimization methods have been proposed in the literature, where they can be classified according to the introduction of fuzzy set theory in: (a) representation of the constraints, and (b) solution method. A typical fuzzy optimization process is described in the next sections.

The main applications of fuzzy optimization in power system problems are: expansion planning [1-5], maintenance scheduling [6,7], unit commitment [8], multi-objective coordination [9-11], and optimal power flow [12-14].

FUZZY OPTIMIZATION BY PSEUDOGOAL FUNCTION

Description of the Process

Usually, optimization problems with a single-real variable are solved using bisection methods, where the main idea is to reduce an initial interval until a required minimum.

Differently from the classical optimization methods, the main idea in fuzzy optimization is to optimize objective function and constraints, simultaneously. In order to determine the optimal point (solution point), both objective function and constraints must be characterize by membership functions and they must be linked by a linguistic conjunction: "and" (for maximization) and "or" (for minimization).

The fuzzy optimization by pseudogoal was proposed by Bellman and Zadeh [21] and the main idea is to satisfy a fuzzy objective function and fuzzy constraints that receive the same treatment, i.e., there is no difference among the objective function and constraints. The first step is a fuzzification process of the objective function, this procedure converts the objective function $f(x_j)$ into a pseudogoal $F(x_j)$ by the following fuzzification process

$$\mu_F(x_j) = \frac{f(x_j) - I}{S - I}$$

where S and I are the maximum and minimum possible values in the feasible interval for the function $f(x_j)$, respectively.

The constraints may also receive the same fuzzification process as above, or they are previously defined as a membership function. In the latter possibility, this definition can represent an expertise (or a linguistic value). For example, using a crisp function, a possible constraint can be $x \leq 3$. The same constraint can be expressed as "the good value is equal or less than 3". A possible complement of this statement may be "it is also acceptable a value not so larger than 3". A possible membership function to represent this linguistic value can be

$$C(x) = \begin{cases} 1 & \text{for } x \leq 3 \\ a + \dfrac{k}{x} & \text{for } x > 3 \end{cases}$$

where k represents how acceptable is the value larger than 3. If the value of k is small (usually, $k < 1$), only values very close to 3 are acceptable; otherwise ($k > 1$), k can represent bigger values for the membership functions. Figure 2 presents an example of these values. The constant a is only a parameter for level adjustment and it is used to turn the membership function to a continuous one.

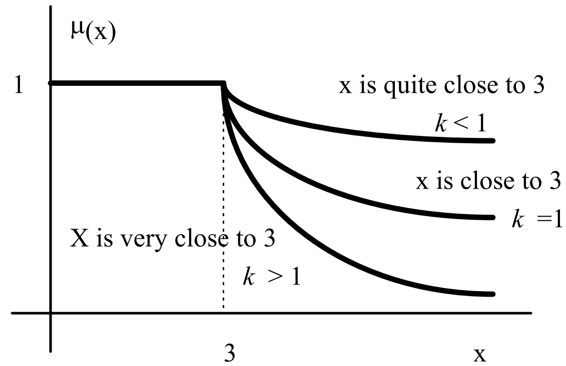


Fig.2 - Possible Membership Functions for a Generic Constraints.

Another usual procedure is the use of fuzzy numbers to define constraints. In classical optimization, intervals define the region to be explored. In fuzzy optimization, this region can be expressed using fuzzy numbers. An example of this procedure is shown in Figure 3, where δ_1 e δ_2 can be defined as the fulfillment (or relaxation) of the constraint.

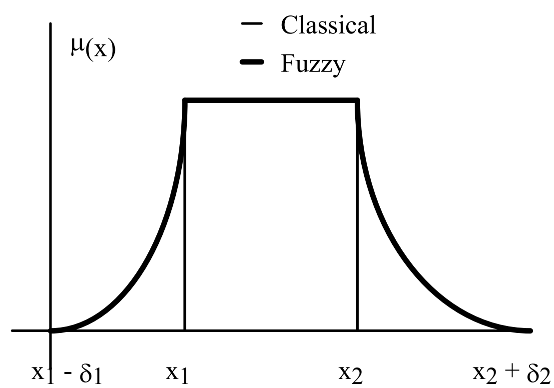


Fig.3 - Classical and Fuzzy Intervals: $[x_1,x_2]$ and $[x_1 + \delta_1, x_2 + \delta_2]$, respectively.

After the fuzzification process, the membership of the optimal function can be found by the aggregation of all constraints and the pseudogoal. In the computation of the fuzzy maximum function, all membership functions are initially merged by the conjunction "and" (intersection of all function, operator: minimum) and then the optimal value (solution) x^* is computed by the operator maximum (i.e., the maximum-minimum value of the membership function). This procedure can be presented by the following sequence, where *G(*x*)* represents the decision function, and $\mu_G(x)$ is its associated membership,

$$\mu_G(\mathrm{x}) = min(C, F)$$

$$\mathrm{x}^* = max\{\mu_G(\mathrm{x})\}$$

In fact, this last operation (maximum) is a defuzzification process, i.e., x^* is the optimal value in the original scale.

In the same way, for the fuzzy minimum function, a sequence can also be structured. Initially, all membership functions are merged by the conjunction "or" (it means the union of all membership functions, operator maximum) and then the optimal value x^* is computed by the operator minimum, as defined by next

$$\mu_G(\mathrm{x}) = max(C, F)$$

$$\mathrm{x}^* = min\{\mu_G(\mathrm{x})\}$$

In the fuzzy optimization process, it is possible to incorporate weights for each constraint and pseudogoal. These weights can represent linguistic hedges in order to modify a membership function (as a linguistic value). Also, other operators (than maximum and minimum) can be used to define relations among constraints and pseudogoal. Sometimes, composite operators must be used for a better definition of the relations [22].

Numerical Example

This section presents a numerical illustrative example on the use of fuzzy optimization for one-single variable. Let be an objective function that represents the following linguistic statement "x must be around 4" and the two constraints: C_1 = "x must be equal or greater than 2 and equal or less than 6", and C_2 = "a good value for x is equal or less than 3 and an acceptable value is not much greater". In this example, the former constraint is a crisp function, while the latter constraint is a fuzzy value. Let's consider the example below.

maximize $f(\mathrm{x}) = 10 - \mathrm{x} - 25/\mathrm{x}^2$

subject to

$$C_1(\mathrm{x}) = \begin{cases} 0 & \text{for x} < 2 \\ 1 & \text{for } 2 \le \text{x} \le 6 \\ 0 & \text{for x} > 6 \end{cases}$$

$$C_2(\mathrm{x}) = \begin{cases} 1 & \text{for x} \le 3 \\ \dfrac{1}{3} + \dfrac{2}{\mathrm{x}} & \text{for x} > 3 \end{cases}$$

The initial step is to compute the pseudogoal *F(*x*)* using the minimum and maximum values of *f(*x*)*:

*f(*x=2*)* = I = 1.75 (minimum value in the interval [2,6])

*f(*x=3.68*)* = S = 4.47 (maximum value in the interval [2,6])

Thus,

$$F(\mathrm{x}) = \frac{f(\mathrm{x}) - \mathrm{I}}{\mathrm{S - I}} = 3.03 - \frac{\mathrm{x}}{2.72} - \frac{9.19}{\mathrm{x}^2}$$

As the constraints have been defined as membership functions, the next step is compute the membership of the decision function *G(*x*)*. This computation is performed using the linguistic conjunction "and" because the objective function and the constraints must be satisfied simultaneously. The result is shown in Figure 4, where the minimum operator has been used. The bold curve is the decision membership function.

The final step is the computation of the optimal value of x^* by the maximum relation of *G(*x*)*. In this case, the maximum value (optimal value) is located in the intersection between the second part of constraint C_2 and the pseudogoal *F(*x*)*. Equaling the two function, the final value of x^* is equal to 3.2.

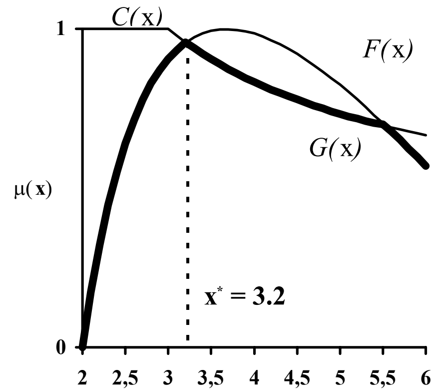


Fig.4 - Computation of Membership Functions.

FUZZY PROGRAMMING

Fuzzy Linear Programming

A classical linear programming can be defined by an optimization of a linear objective function and linear constraints. Usually, this procedure can be represented by the following statements

$$\text{maximize} \quad f(x) = c^T.x$$

$$\text{subject to} \quad Ax \leq b$$
$$x \geq 0$$

where c(n×1), b(m×1), A(m×n) and m<n. The inequality constraints form a feasible region.

The fuzzy linear programming has the same structure of the classical linear programming. The difference between these two approaches is that in the classical approach values and operators are crisp, while, in the fuzzy approach values and/or operators may assume fuzzy characteristics. Examples of this fuzzy transformation may be:

- the operator "maximize" cannot specifically be a search for the optimal but only a "improving of quality",
- the operators ≤ and ≥ can express functions as shown in Figure 2, where for the "belong" crisp region the value of membership is equal to 1, and outside this region, an exponential function defines the membership values, and
- the elements of the vectors b and c and the matrix A can also be a fuzzy definition for a better representation of the real world.

Many contributions have been made in this field, composing the above features [23] and defining new fuzzification and inference processes [24]. Other developments, including duality theory, sensitivity analysis, and integer fuzzy programming can be found in [25].

Fuzzy Dynamic Programming

The idea in classical dynamic programming is to decompose a main problem into several subproblems (one for each variable). Thus, the optimization of each subproblem is divided in a multistage decision process. Here, all operators and values have a crisp meaning. In the same way, a fuzzy dynamic programming can be defined as a fuzzification of all (or part of) these elements. In a well-known fuzzy dynamic programming method, Belmann and Zadeh [21] have proposed to work with fuzzy constraints and fuzzy goals to determine the subgoals of each step of the process, while the transformation function is maintained crisp. An excellent example of the application of fuzzy dynamic programming to power systems is presented in [15].

FUZZY MULTI CRITERIA ANALYSIS

Description of the Problem

During any decision making process, many different factors must be taken into account. These factors can be heuristic or arising from numerical analysis. Usually, the heuristic factors are due to the planner's previous experience and have a non-numerical structure, i.e., they can be better expressed by linguistic values. The problem that planners face in their daily job is how to incorporate these linguistic values into numerical analysis. Commonly, the computational packages do not include the possibility to using non-numerical values. Thus, planners have two possibilities when using this kind of knowledge. One is to put in numbers the linguistic knowledge. The other possibility is to forget this knowledge during the numerical analysis and then, after getting the final result, modify it so as to make an adaptation to take into account the planner expertise.

The problem is that both approaches are not good. In the first one, where planner tries to transform linguistic knowledge into numerical values, much information is lost during this process. For example, if the following statement is to be incorporated: "The distribution feeder A is quite loaded." What is a good numerical value for "quite loaded" ? Two possible ways can be taken; that is, the planner uses a number to define it, for example, 0.80 pu, or he/she can use a percentage, say 90%. Here we also lose information in both transformations. In the first one (the worst transformation), if 0.80 pu relates to a 0.85 pu feeder capacity, the statement does not include information about other numbers around 0.80 pu, for example: 0.78, 0.82, and so on. Each of these expresses the same knowledge and have the same result. On the other hand, the number 0.80 alone can not represent "quite loaded feeder", for example, if the feeder capacity is 1.30 pu.

The second representation of the statement, using a percentage or a range, has the same lost of information problem. Let us assume a "small change" in the percentage number; for a long-term decision-making process, it may result in the same final decision. The problem is that it is very hard to quantify 'what is a small change' in a conventional computational tool.

The other possible approach is to modify the final result in order to take into account the planner expertise. This approach has been commonly used in practical analyses; however, planners have had difficulty in explaining why they need to modify a final value, mainly, if this modification can change the final result order given by the decision process.

Classification of Fuzzy Multi-Criteria Analysis

The classification of fuzzy multi-criteria problems is divided in two main types: multi-objective decision-making and multi-attribute decision-making. In general, the difference between these two approaches is located in the decision space. For the former approach, this space is continuous, and the problem is solved by mathematical programming. For the latter approach, the decision space is discrete, and other approaches have been developed [16,26]. The next subsection presents an algorithm to treat this problem.

Presentation of a Multi-Attribute Decision-Making Algorithm

This algorithm is an extension of Dhar's algorithm, proposed in [17]. Some aspects of data structure representation, inclusion of a new matrix composition and a different fuzzy decision-making process are some modifications and extensions proposed in this algorithm. The original algorithm divides the structure of the problem in alternatives, scenarios and criteria, and its matrix representation. Several facilities are included in the user-interface for an easy accomplishment of the tasks.

The steps of the proposed algorithm are presented as follows:

Step 1: Choose the alternatives to solve the problems and the criteria that will be used in the decision-making process.

Step 2: Create scenarios with fuzzy weights for each criterion and give the conjunctions to compose them.

Step 3: Create a matrix by the combination between scenarios and alternatives for each decision criterion. These matrices must contain information about the relation between each scenario and each alternative in the light of each criterion.

Step 4: Create the fuzzy conditional statements to represent possible data-base knowledge.

Step 5: Obtain, for each matrix of Step 3, the fuzzy set Z_i that is formed by the input weights, according to

$$Z_i = \mu_i(x_{j,k}) / p_{j,k}$$

where i, j and k represent criterion, alternative and scenario, respectively; and $p_{j,k}$ is the weight assigned to the alternative j for a scenario k in a given criterion i.

Step 6: Obtain the fuzzy set L_i formed by the weights $p_{j,k}$ that are assigned to the pertinence matrix which, in turn, is given by the ratio between each weight and the largest value among all weights of the same matrix. The following equations express these value, where Λ represents the largest weight of the matrix,

$$L_i = \mu_\Lambda(x_{j,k}) / p_{j,k}$$

$$\mu_\Lambda(j,k) = p_{j,k} / \Lambda$$

Step 7: Obtain from Z_i and L_i a matrix C_i that is expressed by the equations,

$$C_i = \mu_{C_i}(x_{j,k}) / p_{j,k}$$

$$\mu_{C_i}(x_{j,k}) = \min(\mu_i(x_{j,k}), \mu_\Lambda(x_{j,k}))$$

Step 8: Use MAX, MIN, and algebraic sum operators to compose the fuzzy decision set, according to Step 2.

Step 9: Present the final decision set for each criterion, and the total result.

The Steps 5 to 7 have been proposed by Dhar in his original algorithm. More information about the algorithm to build the fuzzy conditional statements to represent data sets can be found in [18,19].

ILLUSTRATIVE EXAMPLE

Types of Generation System

The expansion strategy of a generation system is to be analyzed, at long term, for a given region. The generation options are hydroelectric plants (H) and nuclear-type thermoelectric plants (N), natural gas (NG), coal (C) and oil fuel (OF). This expansion policy is also intended to be associated to investments in electrical power conservation programs trying to establish, within some scenes, an option scale of generation and conservation measures.

The characterization of each plant, under quantitative and qualitative standpoint, is shown in Table 1. Some data have been obtained from Brazilian Power System (Eletrobrás) Internal Reports. These values are divided in two groups: numerical values and linguistic values. For the generating system there are construction and generation costs; for the electric power there are the "demand reduction cost" and the "saved energy cost", in (US$/kW) and (US$/kWh), respectively. Tables 2 and 3 illustrate these costs for the industrial sector and final uses of electrical power [20].

Table 1 - Quantitative and Qualitative Characteristics of the Generation Systems

	Construct. Cost (US$/kW)	O&M Cost (US$/km /year)	Unity Generation Cost (US$/kWh)	Environmental Costs	Generation Reliability	Ease of Implementation
H	1500	7	0.032	Small	Very High	Small
N	1660	44	0.059	Very High	High	High
NG	1100	22	0.051	Small	Regular	Regular
C	1400	28	0.045	Regular	Regular	Regular
OF	1200	12	0.073	Regular	Regular	Regular

Table 2 - Electrical power conservation costs for industrial sector

	Demand Reduction Cost (US$/kW)	Saved Energy Cost (US$/kWh)
Motors	200-1600	0.02-0.04
Direct Heating (Furnaces)	200-1200	0.02-0.03
Indirect Heating	200-900	0.01-0.02
Electrochemical Processes	200-600	0.01-0.03
Lighting	200-1300	0.02-0.04

Note: Indirect heating includes boiler and water heating.

Table 3 - Comparison of different lighting options

Kind of Lamp	Power (W)	Average Operating Life (hours)	Saved Energy Cost (US$/kWh)	
			A	B
Incandescent Economical (I1)	54	1000	0.027	0.026
Common Tubular Fluorescent (I2)	20	6000	0.031	0.026
Fluorescent Compact (I3)	13	8000	0.060	0.049

Note: A - considering 3 hours/day operation

B - considering 10/hours/day operation

For motors, main electrical power consumer in the industrial sector, it is possible to work in programs of energy conservation which seek, for example, a better used and adaptation in the industrial process (MOTOR 1), the employment of more efficient motors (MOTOR 2) or even the use of varying speed controllers (MOTOR 3) applied to varying torque motors. Each of these options presents different saving energy costs, as shown in Table 4.

Table 4 - Comparison of different costs of the electrical

Kind of Program	Saved Energy Cost (US$/kWh)
Motor 1 (M1)	0.01
Motor 2 (M2)	0.02
Motor 3 (M3)	0.04

Based on the information that we can obtain from Tables 1 to 4, the several technologies aiming at electrical power conservation can be quantitatively and qualitatively characterized within a planning horizon.

Energy Conservation Scenarios and Characteristic Matrices

By attributing a weight from 0 to 10, for example, or a fuzzy linguistic variable, that represents subjectively the importance of each generation plant and the actions of electrical power conservation in the final uses, scenes can be established and the so-called characteristic matrices can be constructed (Table 5).

Table 5 - Electrical Power Conservation Scene in the Planning Horizon

Possible States	Description	Membership Degree
HM	Household Medium	0.3
IM	Industrial Medium	0.4
IH	Industrial High	0.8
CL	Commercial Low	0.2

The characteristic matrices are constructed for the following analysis criteria:

- construction cost or demand reduction cost
- operation and maintenance (O&M) cost;
- generation cost or saved energy cost;
- environmental costs;
- generation reliability or action reliability;
- ease of implementation; and
- usefulness to the entrepreneur.

As an example of this kind of matrix, Table 6 shows the O&M cost characteristic matrix.

Table 6 - Characteristic Matrix - O&M Cost

Alternatives States	HM	IM	IH	CL
H	8	9	10	7
N	2	2	2	2
NG	3	3	4	3
C	2	2	3	2
OF	5	6	7	5
I1	VH	H	H	VM
I2	H	VM	VM	M
I3	H	VM	VM	M
M1	VL	VM	VM	VL
M2	VL	VM	VM	VL
M3	VL	M	M	VL

Calculation of the Fuzzy Decision Set

By using the proposed methodology, the following decision set D is obtained:

D = { (0.6073/H) , (0.4476/N) , (0.6223/NG) , (0.5867/C) , (0.5257/OF) , (0.5692/I1) , (0.5375/I2) , (0.5432/I3), (0.7165/M1) , (0.7032/M2) , (0.5272/M3) }

Thus, for the conditions stated above, the investment strategy in conservation and generation of electrical power is as follows:

1st option: To improve the use and suitability of the motor in the industrial process.

2nd option: To employ more efficient motors in the motors in the process.

3rd option: Hydroelectric generation

4th option: Natural gas thermoelectric generation

5th option: Coal thermoelectric generation

6th option: Nuclear thermoelectric generation

Consider, as an example, that the electric power conservation potential through employment of the 1st option is 8 (TWh) and with the 2nd option is 5 (TWh). Consider also a prediction in the planning horizon of the electrical power market in the order of 60 (TWh). Then, once the 2 first options of conservation are exhausted, there would be a deficit of 47 (TWh). By using the policy of avoiding this deficit only through generation and by considering hydroelectric generation potentials of 25 (TWh), natural gas thermoelectric generation of 10 (TWh) and coal generation of 20 (TWh) yields the following strategy:

Conservation:

13 (TWh), representing 22% of the power demand

Generation:

47 (TWh), representing 78% of the power demand

Conservation actions:

Use 1st and 2nd options

Generation actions:

53% for hydroelectric generation

21% for natural gas thermoelectric generation

26% for coal thermoelectric generation

RESUMO

O uso de técnicas de otimização tem sido importante para reduzir custos e perdas nos sistemas elétricos de potên cia. Um dos pontos mais importantes no processo de otimização é o custo computacional para se achar a melhor solução. Algumas vezes, este custo pode compreender o uso de técnicas para resolver um problema. Sào exemplos de custos computacionais: o número de limições, o número de variáveis e a pequena velocidade de convergência. Este artigo apresenta o desenvolvimento de um processo de otimização difusa. Neste artigo começa com os aspectos teóricos do processo de otimização difusa e então apresenta um exemplo de conservação de energia em sistemas de potência.

PALAVRAS-CHAVE: Modelagem, teoria dos conjuntos difusos, técnicas de linearização.

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